DEPARTMENT OF APPLIED MATHEMATICS,

FACULTY OF ENGG. & TECH.,

M.J.P.R.U., BAREILLY.

Syllabus of M.Sc. (Mathematics)

Session : 2020-21.

M.Sc. (Mathematics)

Syllabus (for Semester System)

The **M.Sc**. **(Mathematics)** course will be of two years duration, each year divided into two Semesters of equal durations. Courses in each Semester will be divided into six papers, details of which are as follows

For M.Sc. I year, First Semester Exams :

For M.Sc. I year, Second Semester Exams

For M.Sc. II year, Third Semester Exams :

PapersIV&V: Any two of the optional papers listed below---

Optional Papers For Third Semester:

For M.Sc. II year, Fourth Semester Exams :

PapersIV&V: Any two of the optional papers listed below---

Optional Papers For Fourth Semester:

Norms for Exams &Evaluation :

1. 50% internal and 50% external evaluation system will be followed.
2. Max. marks in each (Theory/ Oral) paper will be as follows:
   70 marks in End Sem. Exam.
   30 marks in Mid Sem. Exam./Presentation and viva-voce(Internal).

The mid sem. exams will be of internal nature and will be conducted through two tests, each of equal marks such that the first test will cover the first one-third of the syllabus whereas the second test will cover the next one-third of the syllabus. However, if a student fails to appear in one or both the (Mid) tests due to genuine reason (like Medical, etc.) , he or she will be allowed for only one make-up test (of 15 marks only) covering the entire syllabus of the concerned paper).

3. End Sem. Exam of each paper will be of three hours duration whereas each Mid/Make-up test will be of one hour duration. Internal Viva and presentation will be conducted throughout the semesters as per the schedule provided in the time-table.

Notes 1. Permission for optional papers in IV sem. will depend upon the selection of those in III sem., as the only allowable combinations of the papers are-

Papers MA – 634(a) and MA – 644(a)

Papers MA – 634(b) and MA – 644(b)

Papers MA – 634(c) and MA – 644(c)

Papers MA – 634(d) and MA – 644(d)

Papers MA – 634(e) and MA – 644(e)

Papers MA – 634(f) and MA – 644(f)

Papers MA – 634(g) and MA – 644(g)

Papers MA – 634(h) and MA – 644(h)

Papers MA – 634(i) and MA – 644(i)

Papers MA – 634(j) and MA – 644(j).

Papers MA – 634(k) and MA – 644(k).

2 .Teaching of optional papers will be arranged subject to availability of concerned experts/faculty existing in department.

M.Sc. I year Mathematics, First Semester.

Paper -I: MA-511, (Advanced Abstract Algebra -I)

Note : Any five out of eight questions are to be attempted.

Groups: Conjugacy relation, Normalizer, class equation of a finite group, Direct Product of groups, Sylow's theorems, Sylow p-subgroups, Structure Theorem for finite Abelian groups, Normal and subnormal series, Composition series, Solvable groups, Nilpotent groups, Jordan-Holder theorem (3 questions)

Rings: Ideals and Quotient rings, Fields of Quotients and embedding theorems, Divisibility in a commutative ring, Principle ideal domain, Associates, Concept of H.C.F. and L.C.M. in integral domain, Euclidean domain, Unique Factorization domain. (2 questions)

Fields: Extension fields-finite, algebraic and transcendental extensions, Separable and inseparable extensions, Normal extensions, Perfect fields, Finite fields, Primitive elements, Algebraically closed fields. Automorphism of extensions; Fundamental theorem of Galois Theory, Solution of polynomial equations by radicals. (3 Questions)

References :

1. I.N. Herstein, Topics in Algebra, Vikas Publishing House, New Delhi (2^{nd} edition), 1975

2. Surjeet Singh and QuaziZameeruddin, Modern Algebra, Vikas Publishing House, New Delhi (7^{th} Edition) 1997

3. C, Musili, Introduction to Rings and Modules, Narosa Publishing House, New Delhi (2^{nd} edition) 1994

4. N.S. Gopalkrishnan, Commutative Algebra, Oxonian Press, New Delhi, 1984

5. VivekSahai and VikasBist, Algebra, Narosa publishing House, New Delhi, 1999.

6. N.Jacobson, Basic Algebra, Vols. I & II, W.H. Freeman, 1980 (also published by Hindustan Publishing co.)

7. S.Lang, Algebra, (3rd edition), Addison- Wesley, 1993

8. M.Artin, Algebra, Prentice- Hall of India, 1991

9. S.Kumaresan, Linear Algebra: A Geometric Approach, Prentice-Hall of India, 2000.

10. J.Stewart, Galois Theory (2nd edition), Chapman and Hall, 1989.

M.Sc. I year Mathematics, First Semester.

Paper -II : MA-512, (Real Analysis)

Note : Any five out of eight questions are to be attempted.

Definition and existence of Riemann-Stieltjes integral, Properties of the integral. Integration and differentiation. The fundamental theorem of calculus, investigation of vector valued functions, Rectifiable curves. (2 ½ questions)

Functions of several variables, Linear transformation, Derivatives in an open subset of R^n, Chain rule, partial derivatives, Interchange of the orders of differentiation, Derivatives of higher orders, Taylor's Theorem, Inverse function theorem, Implicit function theorem, Jacobians, Extremum problems, problems with constraints, Lagrange's multiplier method. Differentiation of integrals, Partition of unity, Differential forms. Stoke's theorem. (3 questions)

Lebesgue outer measure, Measurable sets. Regularity, Measurable functions, Borel and Lebesgue measurability, Non-measurable sets. Integration of Non-negative functions. The general integral, Integration of series, Riemann and Lebesgue integrals. (2 ½ questions)

References :

1. Walter Rudin, Principles of Mathematical Analysis. (3rd edition). McGraw-Hill Kogakusha, 1976. International Student edition.
2. T. M. Apostol, Mathematical Analysis. Narosa, Publishing House, New Delhi, 1985.
3. P. K. Jain and V. P. Gupta. Lebesgue Measure and Integration. New Age International(P) Ltd., New Delhi, 1986 (Reprint 2000).
4. H.L. Royden, Real Analysis, Macmillan publishing co. Inc. (4th edition). New York, 1993.
5. Walter Rudin, Real and Complex Analysis, McGraw-Hill Book Co., 1966.
6. I. P. Natanson, Theory of Functions of a Real Variable, Vol. 1, FrederickUngar Publishing co., 1961.

M.Sc. I year Mathematics, First Semester.

Paper-III: MA – 513, (Topology- I)

Note : Any five out of eight questions are to be attempted.

Countable and uncountable sets. Infinite sets and the Axiom of Choice. Cardinal number and its arithmetic. Schroeder-Bernstein theorem. Cantor's theorem and the continuum hypothesis. Zorn's lemma. Well-ordering theorem.(2 questions)

Definition and examples of topological spaces. closed sets, closure, Dense subsets, Neighbourhoods, Interior, exterior and boundary, accumulation points and derived sets. Bases and sub-bases, subspaces and relative topology. (2½ questions)

Alternate methods of defining a topology in terms of Kuratowski Closure Operator and Neighbourhood Systems. Continuous functions and homeomorphism. First and second Countable spaces. Lindelof's theorems. Separable spaces. Second Countability and Separability.(2 ½questions)

Separation axioms T_0, T_1, T_2, their characterizations and basic properties. (1 question)

References :

1. James, R. Munkers. Topology, A first Course. Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. J. Dugundji, Topology, Allyn and Bacon 1966 (Reprinted in India by Prentice Hall of India Pvt. Ltd.)
3. George, F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill Book Company, 1963.
4. K. D.Joshi, Introduction to General Topolgy, Wiley Eastern Ltd., 1983.
5. J. Hocking and G. Young, Topolgy, Addison-Wesley, Reading, 1961.
6. J. L. Kelley, General Topolgy, Van Nostrand, Reinhold Co., New York, 1995.
7. L. Steen and J. Seebach, Counter Examples in Topolgy, Holt, Rinchart and Winston, New York, 1970.
8. W., Thom, Topological Structures, Holt, Rinchart and Winston, New York, 1996.
9. N. Bourbaki, General Topology, Polish Scientific Publishers, Warszawa, 1977.
10. W. J. Pervin, Foundations of General Topolgy, Academic Press Inc. New York, 1964.

11. E. H. Spanier, Algebraic Topology, McGraw Hill, New York, 1966.
12. S. Willard, General Topolgy, Addison-Wesely, Reading, 1970.
13. Crump, W. Baker, Introduction to Topology, Wn. C. Brown Publisher, 1991.
14. Sze-Tsen Hu, Elements of General Topolgy, John Wiley and sons, New York, 1963.
15. D. Bushaw, Elements of General Topology, John Wiley and Sons, New York, 1963.
16. M. J.Manisfield, Introduction to Topology, D. Van Nostrand Co., Inc., Boston, 1962.
17. B. Medelson, Introduction to Topolgy, Allyn and Bacon, Inc., Boston, 1962.
18. C. Berge, Topological Spaces, Macmillan Company, New York, 1963.
19. S. S. Coirns, Introductory Topology, Ronald Press, New York, 1961.
20. Z. P. Mamuzic, Introduction to General Topology P, Noordhoff Ltd. Groningen, 1963.
21. K. K. Jha, Advanced General Topology, Nav Bharat Prakashan, Delhi.

M.Sc. I year Mathematics, First Semester.

Paper- IV: MA – 514, (Tensor Algebra and Differential Geometry-I)

Note : In all five out of eight questions are to be attempted selecting at least one from each section .

Section A :

1. Tensor Algebra:

n-dimensional vector space, co-ordinate systems and their transformation laws,Contravariant and covariant vectors and tensors. Metric tensor and its associate tensor. Mixed tensor, Kronecker deltas. Symmetric and skew-symmetric tensors. Addition, subtraction, Scalar multiplication, inner and outer products of tensors. Process of contraction, Quotient law. (1½ questions)

Chirstoffel symbols and their co-ordinate transformation laws. Covariant differentiation. Gradient, divergence and curl. (1½ questions).

Section B:

Differential Geometry I

Curves in space :

3-dimensional Euclidean space, parametric representation of a curve and a surface, Linear element of a curve. Tangent to a curve, Osculating plane, contact of a surface with a curve, curvature and principal normal, circle of curvature, centre and radius of curvature. Binomial and torsion, plane curve. Frenet-Serret formulae. Helices.(3 questions)

Locus of centre of Curvature. Osculating sphere, Locus of centre of spherical curvature, involutes and evolutes of a curve. Co-ordinates in terms of arc length parameter, Intrinsic equation of curve. (2 questions).

References :

1. Lal, Bansi and Arora, Sanjay: Three Dimensional Differential Geometry, Atma Ram and Sons, Delhi, 1989.
2. Mishra, R. S.: A course in Tensors with applications to Riemannian Geometry. Pothishala Pvt. Ltd., Allahabad, 1965.
3. Singh, H. D. and Singh, P. K.: Differential Geometry, Ram Prasad & Sons. Agra.

4. Sinha, B. B., Differential Geometry : An Introduction, ShyamPrakashanMandir, Allahabad, 1978.
5. Weatherburn, C. E. : A Introduction to Tensor Calculus and Reimannian Geometry, Cambridge University Press, London, 1942 and Radha Publishing House, Calcutta (Indian Edn., 1995).
6. Weatherburn, C. E. Differential Geometry of three Dimensions. Vol-1. Scientific Book Co.,Patna, 1955. Khosla Publishing House and Radha Publishing House, Calcutta, 1988.
7. Eisenhart, L. P., Differential Geometry with the use of Tensors, Princeton University Press, New Jersey, 1949.
8. Willmore, T. J. : Differential Geometry, OxfordUniversity Press. London, 1959 and Indian X1 Edn., New Delhi, 1993.

M.Sc. I year Mathematics, First Semester.

Paper-V: MA – 515, (Operations Research -I)

Note: Any five out of eight questions are to be attempted.

Operations Research and its Scope, Necessity of Operations Research in Industry. (1 question)

Linear Programming-Simplex Method. Theory of the Simplex Method. Duality and Sensitivity Analysis. Other Algorithms for Linear Programming. Dual Simplex Method. Parametric Linear Programming. Upper Bound Technique, interior Point Algorithm, Linear Goal Programming, Introduction of Non Linear Programming. (3 questions)

Transportation and Assignment Problems (1+1questions)

Network Analysis-Shortest Path problem, Minimum Spanning Tree Problem, Maximum Flow Problem, Minimum Cost Flow Problem, Network Simplex Method. Project Planning and Control with PERT-CPM. (2 questions)

References :

1. F. S. Hiller and G. J. Liberman, Introduction to Operations Research (Sixth Edition), McGraw Hill International Edition, Industrial Engineering Series, 1995 (This book comes with a CD containing tutorial software)
2. G. Hadley, Linear Programming, Narosa Publishing House, 1995.
3. G. Hadley, Non-linear and Dynamic Programming, Addison-Wesley, ReadingMass.
4. Mokhtar, S. Bazaraa, John, J. Jarvis and Hanif, D. Sherali, Linear Programming and Network Flows, John Wiley and Sons, New York, 1990.
5. H. A. Taha, Operations Research : An Introduction, Macmillan Publishing Co. Inc., New York.
6. KantiSwarup, P. K. Gupta and Man Mohan, Operations Research, Sultan Chand & Sons, New Delhi.
7. LINDO Systems Products (Visit website http://www.lindo.com/productsf.html)

M.Sc. I year Mathematics, Second Semester.

Paper -VI : MA-516

Presentation and Viva-Voce-I

A : Viva and Presentation of assigned / selected problem /topic by each student in each of the earlier five papers to be evaluated internally throughout the semester

Total Marks : 6x5=30

B :Viva –Voce concerning all other five papers of the semester to be evaluated externally

Total Marks : 70

M.Sc. I year Mathematics, Second Semester.

Paper -1: MA-521, (Advanced Abstract Algebra- II)

Note : Any five out of eight questions are to be attempted.

Vector Spaces: vector subspaces, sum & direct sum of subspaces, linear span, Linear dependence, independence & properties, basis, Dimension of vector spaces. Finite dimensional Vector spaces, Extension theorem for bases, existence of a complementary subspaces of finite dimensional vector space, Quotient space & it's dimension, linear transformation (3 Questions).

Modules: Cyclic modules, simple modules, semi-simple modules, Schur's Lemma, Free modules, Noetherian and Artiman modules and rings. Hilbert basis theorem. Wedderbum- Artin theorem, Uniform modules, Primary modules and Noether-Lasker theorem. (2 Questions)

Canonical forms: Similarity of linear transformations, invariant subspaces, Reduction to triangular forms. Nilpotent transformations, index of Nil-potency, invariant of Nilpotent transformations, Primary decomposition theorem, Jordan Blocks and Jordan forms (3 questions).

References :

1. I.N. Hernstein, Topics in Algebra, Vikas Publishing House, New Delhi (2^{nd} edition), 1975.

2. Surjeet Singh and QuaziZameeruddin, Modern Algebra, Vikas Publishing House, New Delhi (7^{th} Edition) 1997.

3. C, Musili, Introduction to Rings and Modules, Narosa Publishing House, New Delhi (2^{nd} edition) 1994.

4. N.S. Gopalkrishnan, Commutative Algebra, Oxonian Press, New Delhi, 1984.

5. VivekSahai and VikasBist, Algebra, Narosa publishing House, New Delhi, 1999.

6. N.Jacobson, Basic Algebra, Vols. I & II, W.H. Freeman, 1980 (also published by Hindustan Publishing co.).

7. S.Lang, Algebra, (3^{rd} edition), Addison- Wesely, 1993.

8. M.Artin, Algebra, Prentice- Hall of India, 1991.

9. S.Kumaresan, Linear Algebra: A Geometric Approach, Prentice-Hall of India, 2000.

10. J.Stewart, Galois Theory (2^{nd} edition), Chapman and Hall, 1989.

M.Sc. I year Mathematics, Second Semester.

Paper –II: MA-522, (Complex Analysis)

Note : Any five out of eight questions are to be attempted.

Complex integration, Cauchy-Goursat theorem, Cauchy's integral formula. Higher order derivatives, Morera's theorem. Cauchy's inequality and Liouville's theorem. The fundamental theorem of algebra, Taylor's theorem, Maximum modulus principle, Schwarz lemma. Laurent's series, Isolated singularities. Meromorphic functions, The argument principle. Rouchi's theorem, Inverse function theorem. (3½ questions)

Residues, Cauchy's residue theorem, Evaluation of integrals, Branches of many valued functions with special reference to arg z, log z and z^a (2½ question)

Bilinear transformations, their properties and classification. Definition and examples of conformal mappings. (1 question)

Power series, convergence of power series, Integration and differentiation of power series. (1 question)

References :

1. J. B. Conway, Functions of one complex variable. Springer-Verlag, International Student edition, Narosa Publishing House, 1980.
2. L. V. Alhfors, Complex Analysis, McGraw-Hill, 1979.
3. Walter Rudin, Real and Complex Analysis, McGraw-Hill Book Co., 1966.
4. T. Pati, Functions of Complex variable, Pothishala Pvt. Ltd. Allahabad, 1986.
5. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

M.Sc. I year Mathematics, Second Semester.

Paper –III: MA – 523, (Topology-II)

Note: Any five out of eight questions are to be attempted

Separation axiomsT_2, T_3, $T_{3.5}$, T_4,T_5;spaces their characterizations and basic properties. Urysohn's lemma. Tietze extension theorem. (2 questions)

Compactness. Continuous functions and compact sets. Basic properties of compactness. Compactness and finite intersection property. Sequentially and countably compact sets. Local compactness and one point compactification. Stone-Cech compactification. Compactness in metric spaces. Equivalence of compactness, countable compactness and sequential compactness in metric spaces. (2½ questions)

Connected spaces. Connectedness on the real line. Components. Locally connected spaces.(1½ questions)

Tychonoff product topology in terms of standard sub-base and its characterizations. Projection maps. Separation axoms and product spaces. Connectedness and product spaces. Compactness and product spaces (Tychonoffs theorem). Countability and product spaces. Embedding and metrization. Embedding lemma and Tychonoff embedding.Urysohnmetrization theorem [2 questions].

References :

1. James, R. Munkres. Topology, A first Course. Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. J. Dugundji, Topology, Allyn and Bacon 1966 (Reprinted in India by Prentice Hall of India Pvt. Ltd.)
3. George, F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill Book Company, 1963.
4. K. D.Joshi, Introduction to General Topology, Wiley Eastern Ltd., 1983.
5. J. Hocking and G. Young, Topology, Addison-Wesley, Reading, 1961.
6. J. L. Kelley, General Topology, Van Nostrand, Reinhold Co., New York, 1995.

7. L. Steen and J. Seebach, Counter Examples in Topology, Holt, Rinchart and Winston, New York, 1970.
8. W., Thom, Topological Structures, Holt, Rinchart and Winston, New York, 1996.
9. N. Bourbaki, General Topology, Polish Scientific Publishers, Warszawa, 1977.
10. W. J. Pervin, Foundations of General Topology, Academic Press Inc. New York, 1964.
11. E. H. Spanier, Algebraic Topology, McGraw Hill, New York, 1966.
12. S. Willard, General Topology, Addison-Wesley, Reading, 1970.
13. Crump, W. Baker, Introduction to Topology, Wn. C. Brown Publisher, 1991.
14. Sze-Tsen Hu, Elements of General Topology, John Wiley and sons, New York, 1963.
15. D. Bushaw, Elements of General Topology, John Wiley and Sons, New York, 1963.
16. M. J.Manisfield, Introduction to Topology, D. Van Nostrand Co., Inc., Boston, 1962.
17. B. Medelson, Introduction to Topology, Allyn and Bacon, Inc., Boston, 1962.
18. C. Berge, Topological Spaces, Macmillan Company, New York, 1963.
19. S. S. Coirns, Introductory Topology, Ronald Press, New York, 1961.
20. Z. P. Mamuzic, Introduction to General Topology P, Noordhoff Ltd. Groningen, 1963.
21. K. K. Jha, Advanced General Topology, Nav Bharat Prakashan, Delhi.

M.Sc. I year Mathematics, Second Semester.

Paper – IV: MA – 524, (Differential Geometry-II)

Note : Any five out of eight questions are to be attempted .

Surfaces in E_3:

Various forms of a surface, explicit form. Gaussian and Monge's forms. Different types of surfaces, right helicoid, conicoid, surface of revolution; Tangent plane to a surface. One parameter family of surfaces, their characteristic curve and envelope. Ruled surfaces; Developable and skew-surfaces, properties of developable. Developable associated with space-curves. (1½ questions)

Intrinsic Geometry of Surfaces:

Curvilinear co-ordinates. Fundamental magnitudes of first order. Christoffel symbols. Direction on a surface. Angle between two directions. Orthogonality and parallelism of two directions determined by a quardratic equation. Inclinations of direction with parametric curves. Normal to a surface. Fundamental magnitudes of second order. Derivatives of unit normal to a surface.(1½ questions).

Normal and oblique sections of a surface and their curvatures. Meusnier's theorem, Normal curvature. Principal curvatures. Principal directions, First and second curvatures. Minimal surface. Umbilic point and umbilical surfaces. Lines of curvatures. Joachimsthal's theorem. Rodrigues formula. Parametric curves as lines of curvature. Euler's formula for normal curvature, Catenoid as the only real minimal surface of revolution. Developables associated with lines of curvature. (2 questions)

Asymptotic Lines and Geodesics:

Asymptotic lines, Beltrami-Enneper's theorems. Curvature of asymptotic lines. Geodesics, Euler-Lagrange conditions. Differential equations of geodesics. Existence theorem. Properties of geodesics. Parametric curves as geodesics. Torsion of a geodesic, Bonnet's theorem, Joachimsthal's theorem. Geodesic curvature of curve. (3 questions)

References :

1. Lal, Bansi and Arora, Sanjay : Three Dimensional Differential Geometry, Atma Ram and Sons, Delhi, 1989.
2. Singh, H. D. and Singh, P. K. : Differential Geometry, Ram Prasad & Sons. Agra.
3. Sinha, B. B., Differential Geometry : An Introduction, ShyamPrakashanMandir, Allahabad, 1978.
4. Weatherburn, C. E. Differential Geometry of three Dimensions. Vol-1. Scientific Book Co. Patna, 1955. Khosla Publishing House and Radha Publishing House, Calcutta, 1988.
5. Eisenhart, L. P., Differential Geometry with the use of Tensors, Princeton University Press, New Jersey, 1949.
6. Willmore, T. J. : Differential Geometry, OxfordUniversity Press. London, 1959 and Indian X1 Edn., New Delhi, 1993.

M.Sc. I year Mathematics, Second Semester.

Paper –V: MA – 525, (Operations Research- II)

Note : Any five out of eight questions are to be attempted.

Dynamic Programming, Deterministic and Probabilistic Dynamic Programming. (1½ question)

Game Theory : Two-Persons Zero-Sum Games. Games with Mixed Strategies, Graphical Solution. Solution by Linear Programming. (2 questions).

Elementary inventory Models. Inventory models with price breaks. (1½ questions)

Integer Programming, Branch and Bound Technique. Applications to Industrial Problems, Optimal Product mix and activity levels. Petroleum refinery operations. Blending problems. Economic Interpretation of dual linear programming problems. Input-Output analysis. Leontief system, Indecomposable and Decomposable Economics. (3 questions)

References :

1. G. Hadley, Non-linear and Dynamic Programming, Addison-Wesley, ReadingMass.
2. H. A. Taha, Operations Research : An Introduction. Macmillan Publishing Co. Inc., New York.
3. KantiSwarup, P. K. Gupta and Man Mohan, Operations Research. Sultan Chand & Sons, New Delhi.
4. S. S. Rao, Optimization Theory and Applications, Wiley Eastern Ltd., New Delhi.
5. Prem Kumar Gupta and D. S. Hua, Operations Research – An Introduction, S. Chand & Company Ltd., New Delhi.
6. N. S. Kambo, Mathematical Programming Techniques, Affiliated East-West Press Pvt. Ltd., New Delhi, Madras.

M.Sc. I year Mathematics, Second Semester.

Paper -VI : MA-526

Presentation and Viva-Voce-II

A : Viva and Presentation of assigned / selected problem /topic by each student in each of the earlier five papers to be evaluated internally throughout the semester

Total Marks : 6x5=30

B : Viva –Voce concerning all other five papers of the semester to be evaluated externally

Total Marks : 70

M.Sc. II year Mathematics, Third Semester.

Paper-I : MA – 631, (Integration Theory and Functional Analysis-I)

Note : In all five out of eight questions are to be attempted selecting at least one from section A.

Section A :Integration Theory

Signed measure, Hahn decomposition theorem, mutually singular measures. Radon-Nikodym theorem. Lebesgue decomposition. Reisz representation theorem. Extension theorem (Caratheodory), Lebesgue-Stielties integral, product measures, Fubini's theorem. Differentiation and Integration. Decomposition into absolutely continuous and singular parts. (3 questions)

Section B :Functional Analysis

Normed linear spaces. Banach spaces and examples. Quotient space of normed linear spaces and its completeness. Equivalent norms. Reisz Lemma. Basic properties of finite dimensional normed linear spaces and compactness. Weak convergence and bounded linear transformations. Normed linear spaces of bounded linear transformations. Uniform boundedness theorem and some of its consequences. Open mapping and closed graph theorems. (5 questions)

References :

1. H. L., Royden, Real Analysis, Macmillan Publishing Co. Inc. New York. 4th Edition, 1993.
2. G. de, Barra, Measure Theory and Integration. Wiley Eastern Limited, 1981.
3. P. R. Halmos, Measure Theory, Van Nostrand, Princeton, 1950.
4. Inder K. Rana, An Introduction to Measure and Integration, Narosa Publishing House. Delhi, 1997.
5. Edwin, Hewitt and Kort Stromberg, Real and Abstract Analysis, Springer Verlag, New York.
6. G. Bachman and L. Narici, Functional Analysis, Academic Pres, 1966.
7. C. Goffman and G. Pedrick, First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.

8. P. K. Jain, O. P. Ahuja and Khalil Ahmad, Functional Analysis, New Age International (P) Ltd. & Wiley Eastern Ltd., New Delhi, 1997.
9. K. K. Jha, Functional Analysis, Student's Friends, 1986.
10. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, New York, 1978.
11. B. K. Lahiri, Elements of Functional Analysis, The World Press Pvt. Ltd., Calcutta, 1994.
12. B. Chaudhary and Sudarsan Nanda, Functional Analysis with Applications, Wiley Eastern Ltds., 1989.
13. B. V. Limaye, Functional Analysis, Wiley Eastern Ltd.
14. A.H. Siddqui, Functional Analysis with Application, Tata Mac Graw Hill Publishing Company Ltd., New Delhi.
15. A. Wilansky, Functional Analysis, Blasidell Publishing Co., 1964.
16. K. Yosida, Functional Analysis. 3rd edition, Springer-Verlag, New York, 1971.
17. G. F. Simmons, Introduction to Topology and Modern Analysis. McGraw Hill Book Company, New York, 1963.

M.Sc. II year Mathematics, Third Semester.

Paper- II :MA-632, (Theories of Differential Equations)

Note: Any five out of eight questions are to be attempted.

Existence and uniqueness Theorem of Homogeneous and Non Homogeneous equations with constant coefficients. Theory of equations with variable coefficients. Method of variation of parameters and the formula for particular integral in terms of Wronskian.(2 questions)

Series solution of second order linear differential equations near ordinary point. Singularity and the solution in the neighborhood of regular singular point. Euler equation and Frobenius method, solution of Legendre, Bessel, Hypergeometric, Hermite and Laguerre differential equations. (3 questions)

Green's functions and the solution of boundary value problems in terms of Green's functions. Concepts of stability, asymptotic stability and instability of a solution of the autonomous system.

$$\frac{dx}{dt} = F(x,y), \qquad \frac{dy}{dt} = G(x,y) \text{(3 questions)}$$

References:

1. Earl A. Codington, An Introduction to Ordinary Differential Equations.
2. Elementary Differential Equations and Boundary value problems.
3. D.A. Murray, Introductory Course on Differential Equations. Orient Longman (India) 1967.
4. A.R. Forsyth. A Trestise on Differential Equations, Macmillan & Co. Ltd., London.
5. IN. Sneddon, Special Functions.

M.Sc. II year Mathematics, Third Semester.

Paper – III: MA-633, (Analytical Dynamics)

Note: Any five questions out of eight are to be attempted.

Generalized co-ordinates. Holonomic and Non- holonomic systems, Scleronomic and Rheonomic systems. Generalized potential. Lagrange's equations of first kind. Lagrange's equations of second kind. Uniqueness of solution. Energy equation for conservation fields.(2 question)

Hamilton's variables, Donkin's Theorem. Hamilton canonical equations. Cyclic co-ordinates. Routh's equations. Poisson's brackets. Poisson's identity, Jacobi-Poisson Theorem. Motivating problems of Calculus of variations, shortest distance, Minimum surface of revolution.

Brachistochrone problem. Isoperimetric problem. Geodesic, fundamental lemma of calculus of variations. Euler's equation for one dependent function and its generalization to (i) 'n' dependent functions, (ii) higher order derivatives. Conditional extremum under geometric constraints and under integral constraints. (3 questions)

Hamilton's Principle : Principle of least action. Poincare Carton Integral invariant. Whittaker's equations, Jacobi equations. Statement of Lee Hwa Chung's Theorem (1½ questions)

Hamilton-Jacobi equation. Jacobi Theorem. Method of separation of variables. Lagrange Brackets. Condition of canonical character of a transformation in terms of Lagrange brackets and Poisson brackets. Invariance of Lagrange Brackets and Poisson brackets under canonical transformations. (1½ questions)

References:

1. A. S. Ramsey, Dynamics Part II, The English Language Book Society and CambridgeUniversity Press, 1972.
2. F. Gantmacher, Lecturers in Analytic Mechanics, MIR Publishers, Moscow, 1975.
3. H. Goldstein, Classical Mechanics (2nd edition), Narosa Publishing House, New Delhi.
4. Narayan Chandra Rana and Pramod Sharad Chandra Joag, Classical Mechanics, Tata McGraw Hill, 1991.

5. Louis, N. Hand, Janet D. Finch, Analytical Mechanics, Cambridge University Press, 1998.
6. I. M. Gelfand and S. V. Fomin, Calculus of variations, Prentice Hall.

M.Sc. I Iyear Mathematics, Third Semester.

Paper- IV & V (a) :MA-634 (a), (Advanced Fluid Dynamics-I) (Optional)

Notes: Any five questions are to be attempted out of eight. .

General Theory of Stress and Strain :

Newton's law of viscosity. Body and surface forces. Stress vector and Stress tensor. State of stress at a point. Symmetry of stress tensor : Plane stress, Principal stresses and principal directions. Principal stresses and principal directions of stress tensor. (1 question)

Nature of strain, Normal strain. Shearing strain. Transformation of strain components. Equations for Newtonian (viscous) fluid. Relation between stress and rate of strain. Stokes law of viscosity. Translation, rotation and rate of deformation. (2 questions)

Navier-stokes equation and the energy equation : Navier-stokes equation of motion for a viscous fluid. The energy equation. Equation of state for perfect fluid. Energy dissipation due to viscosity.(2 questions)

Laminar flow of Viscous incompressible fluids : Steady laminar flow between two parallel plates. Plane Couette flow, Generalized plane Couette-flow, Plane Poiseuille flow. Flow through a circular pipe, the Hagen-Poiseuille flow. Laminar steady flow between two coaxial circular cylinders. Laminar flow between two concentric rotating cylinders, Steady flow of viscous incompressible viscous fluid. Diffusion of vorticity. (3 questions)

References:

1. Allen Jeffery – Magnetohydrodynamics (Oliver & Boyd).
2. P. C. Kendell and C. Plumton – Magnetohydrodynamics with hydrodynamics – Vol 1 (Pergamon Press).
3. F. Chorlton – A Text Book of Fluid Dynamics.
4. M. D. Raisinghania& R.S. Agarwal – Advanced Hydrodynamics & Fluid Dynamics.

M.Sc. II year Mathematics, Third Semester.

Paper-IV & V (b):MA-634(b), (Approximation Theory-I) (Optional)

Notes: Any five questions out of eight are to be attempted.

Theorem of Weierstrass for algebraic and trigonometric polynomials. Approximation by integral operators. Korovkin theorem. Stone – Weierstrass theorem. (1½ questions)

Linear Operators, Interpolation of operators, Hardy-Littlewood – Polya Theorem, Moduli of continuity. Moduli of smoothness, Marchaud's inequalities. (1½ questions)

Spehitz spaces, Best approximation, Existence, Kolomogorov's Theorem, Haar systems. (1½ questions)

Uniqueness of best approximation in C(A), Chebychev theorem, Chebyshev polynomials, Strong unicity, Remez algorithm for computation of best approximation. (1½ questions)

Best approximation in Lp ($1<p<\infty$), Chebyshev polynomials of second kind, Polya and Descartes systems. Inequalities of Bernstein, Szego and Markov. (2 questions)

References:

1. R. A. Devore and G. G. Lorentz. Constructive Approximation, Springer – Verlag, 1993.
2. M. J. D. Powell : Approximation Theory and methods, Cambridge University Press, 1981.
3. G. G. Lotentz. Approximation of functions, First ed. Holt Rinchart and Winston, New York, 1966.
4. E. W. Cheney, Introduction to Approximation theory,AMS/Chelsea publication, 1968.

M.Sc. II year Mathematics, Third Semester.

Paper- IV & V (c):MA-634(c), (Tensors and General Relativity-I) (Optional)

Note: Any five questions are to be attempted selecting at least one from each section.

Section - A

Tensors:

Transformation of coordinates, Tensors, Algebra of Tensors, Symmetric and skew symmetric Tensors, Contraction of tensors and quotient law.

Riemannian metric, Parallel transport, Christoffel Symbols, Covariant derivatives, Intrinsic derivatives and geodesics. Riemann-Christoffel curvature tensor and its symmetry properties. Bianchi identities and Einstein tensor. (3 questions)

Section – B

General Relativity

Review of the special theory of relativity and the Newtonian Theory of gravitation, Principles of equivalence and general covariance, geodesic principle, Newtonian approximation of relativistic equations of motion. Einstein's field equations and its Newtonian approximation. (2 questions)

Schwarzschild external solution and its isotropic form, Planetary orbits and analogues of Kepler's Laws in general relativity. Advance of perihelion of a planet, Bending of light rays in gravitational field, gravitational red-shift of spectral lines. Radar echo delay. (3 questions)

References:

1. C. E. Weatherburn. An Introduction of Riemannian Geometry and Tensor Calculus, CambridgeUniversity, Press, 1950.
2. H. Stephani, Genereal Relativity. An Introduction of the theory of the gravitational field. CambridgeUniversity Press, 1982.
3. A. S. Eddington, The Mathematical Theory of Relativity, CambridgeUniversity, Press, 1965.

4. J.V. Narlikar, General Relativity, and Cosmology. The Macmillan Company of India Ltd., 1978.
5. R. Adler, M. Bazin, M. Schiffer, Introduction to General Relativity, McGraw Hill Inc., 1975.
6. B. F. Shutz, A first course in general relativity, Cambridge University Press, 1990.
7. S. R. Roy and Raj Bali, Theory of Relativity, Jaipur Publishing House, Jaipur, 1987.

M.Sc. II year Mathematics, Third Semester.

Paper- IV & V (d):MA-634(d), (Information Theory-I) (Optional)

Note: Any five out of eight questions are to be attempted.

Measures of Information – Axioms for a measure of uncertainty. The Shannon entropy and its properties. Joint and conditional entropies. Transformation and its properties. (1 question)

Noiseless coding – Ingredients of noiseless coding problem, Uniquely decipherable codes.

Necessary and sufficient condition for the existence of instantaneous codes. Construction of optional codes. (1 question)

Discrete Memoryless Channel, Classification of channels, information processed by a channel, calculation of channel capacity, decoding schemes. The ideal observer. The fundamental theorem of information theory and its strong and weak converses. (2 questions)

Continuous Channels –The time discrete Gaussian channel. Uncertainity of an absolutely continuous random variable. The converse to the coding Theorem for time discrete Gaussian Channel. The time continuous Gaussian Channel, Band-limited channels. (2 questions)

Some imuitive properties of a measure of entropy-symmetry, normalization, expansibility,boundedless, recursivity, maximality, stability, additivity, subadditivity, non-negativity, continuity, branching etc. and interconnections among them. Axiomatic characterization of the Shannon entropy due to Shannon and Fadeev. (2 questions)

References:

1. R. Ash, Information Theory, Interscience Publishers. New York, 1965.
2. F. M. Reza, An Introduction of Information Theory, McGraw Hill Book Company Inc., 1961.
3. J. Aczel and Z. Daroczy. On measures of information and their characterizations. Academic Press. New York.

M.Sc. II year Mathematics, Third Semester.

Paper- IV & V (e):MA-634(e), (Mathematics of Finance and Insurance-I) (Optional)

Note: Any five out of eight questions are to be attempted.

Prerequisite – Application of Mathematics in Finance and Insurance.

Financial Derivatives – An introduction : Types of Financial Derivatives, Forwards and Futures; Options and its kinds; and SWAPS. (1 question)

The Arbirage Theorem and Introduction to portfolio Selection and Capital Market Theory : Static and Continuous – Time Model. (1 question)

Pricing by Arbitrage – A Single – Period option pricing Model : Multi- Period Pricing Model – Cox-Ross- Rubinstein Model ; Bounds on Option Prices.

The Ito's Lemma and Ito's Integral. (2 questions)

The dynamics of Derivative Prices – Stochastic differential Equation (SDEs) –Major Models of SDEs : Linear Constant Coefficient SDEs ; Geometric SDEs ; Square Root Process ; Mean Reverting Process and OmsteinUhlenbeek Process. ($2\frac{1}{2}$ questions)

Martingale Measures and Risk-Neutral Probabilities : Pricing of Binomial Options with equivalent Martingale measures. ($1\frac{1}{2}$questions)

References :

1. John, C. Hull, Options, Futures and other Derivatives, Prentice Hall of India Private limited.
2. Sheldon M. Ross, An Introduction of Mathematical Finance, Cambridge University, Press.
3. Saliah, N. Neftci, An Introduction to the Mathematics of financial derivatives, Academic Press, Inc.
4. Robert, J. Elliott and P. Ekkehard Kopp, Mathematics of Financial Markets, Springer, Verlag, New York, Inc.

5. Robert C.Merton, Continuous – Time Finance, Basil Blackwell Inc.
6. C. D. Daykin, T. Pentrikamen and M. Pesonen, Practical Risk theory for Actuaries, Chapman & Hall.
7. Tomasz, Rolski, Hanspter, Schmidli, Volker Schmidt and JozefTengels. Stochastic processes for Insurance and Finance, John Wiley & Sons Limited.

M.Sc. II year Mathematics, Third Semester.

Paper IV & V(f):MA-634(f), (Non-Linear Programming-I)(Optional)

Note: Any five questions out of eight are to be attempted.

The Non-linear programming problem and its fundamental ingredients.

Linear inequalities and the theorems of the alternative Kas Theorem. The Optimality criteria of linear programming, Tucker's lemma and existence Theorems of the alternative convex sets-separation theorems.

Convex and concave functions- Basic properties, some fundamental theorems for convex functions. Generalized Gorden theorem, Bohnenblust-karlin-Shapley Theorem (3 questions).

Saddle point optimality criteria without differentiability. The minimization and the local minimization problems and some basic results, sufficient optimality Frit-John saddlepoint necessary optimality theorem. Fritz John saddle point necessary optimality theorem, Slater's and Karlin's constraint qualifications and their equivalence. The strict constraint qualification, Kuhn-Tucker saddlepoint necessary optimality theorems. (2 questions)

Differentiable convex and concave functions –

Some basic properties, Twice differentiable convex and concave functions. Theorems in cases of strict convexity and concavity of functions.

Optimality Criteria with differentiability –

Sufficient optimality theorems, Fritz-John stationary point necessary optimality theorem. The Arrow-Hurwicz-Uzawa constraint qualification. Kuhn-Tucker stationary point necessary optimality theorem. (3 questions)

References:

1. O. L. Mangasarian, Non-linear Programming, McGraw Hill, New York.
2. Mokhtar S. Bazarra and C. M. Shetty, Non-linear programming, Theory and Algorithms, Wiley, New York.
3. Mordecai Avriel, Non-Linear Programming, Analysis and Methods, Prentice Hall Inc.,Englewood Cliffs, New Jersey.

M.Sc. II year Mathematics, Third Semester.

Paper IV & V(g): MA-634(g), (Riemannian Geometry-I) (Optional)

Note: any five out of eight questions are to be attempted.

1. **Tensor Connexions:**

   Affine connexions Lie Bracket. Pseudo tensorial forms. Torsion and curvature forms. Covariant derivative, torsion and curvature tensors, intrinsic derivative of a vector. Levi Civita's concepts of parallelism. (2 questions)

2. **Riemannian manifold Vn**:

   Riemannian metric, Riemannian connexions, Angle between two vectors. Fundamental theorem of Riemannian geometry, Ricci-identities, RiemannianChristoffel (curvature) Tensor, Riccitensor, Bianchi identities,Laplacian operator, geodesics and Riemannian Coordinates, Riemannian curvature. Schur's theorem. Geodesics and conformal mappings, projective curvature tensor, conformal curvature tensor.(4 questions)

3. **Ricci's coefficients of rotation:** Orthonormal bases, Ricci's coefficients of rotation and the reason for their name, Congruences-Geodesic, normal irrotational and canonical, Gaussian and Riccian Curvature. (2 questions)

References :

1. MISHRA, R.S., A Course in Tensor with Application to Riemannian Geometry, Pothishakka Pvt. Ltd. Allahabad, 1965.
2. Weatherburn, C.E.: An Introduction to Tensor Calculus and Riemannian Geometry, Cambridge University Press, London, 1942 and Radha Publishing House, Calcutta, India, Ed. 1995.
3. Yano, K.,The Theory of Lie Derivatives and its Applications, North Holland Publishing Co.,Amesterdam, 1957.
4. Eisenhart: Riemannian Geometry, Princeton University Press, New jersey, 1927.
5. Schouten, J.A.: Ricci Calculus-An Introduction to Tensor Analysis and its Geometrical Applications, Springer Verlag, Berlin, II Ed. 1954.
6. Willmore, T.J.: An introduction to Differential Geometry, Oxford university Press,London 1954 and 11th Indian Ed.,New Delhi, 1993.

M.Sc. II year Mathematics, Third Semester.

Paper-IV & V(h): MA-634(h), (Space Dynamics-I) (Optional)

Note: Any five out of eight questions are to be attempted.

Basic formulae of a spherical triangle- The two body problem, the motion of the centre of mass. The relative motion, Kepler's equation, Solution by Hamilton-Jacobi theory.

The determination of orbits-Laplace and Gauss Methods. (2 questions)

The three Body problem: general three body problem, restricted three body problem, Jacobi Integral, curves of zero velocity, Stationary solutions and their stability.

The n-body problem- The motion of the center of mass, Classical integrals. Pertubation - Osculating orbit, perturbing forces. Secular and periodicperturbations. Langrange'sPlanetory Equations in terms of perturbing forces and in terms of a perturbed Hamiltonian.

Motion of the moon-the perturbing forces, perturbations of Keplerian elements of the Moon by the Sun. (3 questions)

Flight mechanics- Rocket performance in a vacuum. Vertically ascending paths . Gravity twin trajectories. Multi stage rocket in vacuum. Definitions pertinent to single stage rockets.

Performance limitations of single stage rockets, definitions pertinent to multi stage rockets, Analysis of multistage rockets including gravity. Analysis of multistage rockets neglecting gravity.

Rocket performance with Aerodynamic forces. Short range non-lifting missiles, Ascent of a sounding rocket. Some approximate performance of rocket- powered air- craft. (3 questions)

References :

1. J.M.A. Danby, fundamentals of Celestial Mechanics, The Macmillan Company.1962.
2. E.Finlay, Freundlich ,Celestial Mechanics, The Macmillan Company,1958.
3. Theodore E.Sterne, An Introduction of Celestial Mechanics, Intersciences Publishers,Inc.1960.
4. Arigelo Miele, Flight Mehanics-Vol1, Theory of flight paths, Addison-wesley publishing company, Inc.,1962

M.Sc. II year Mathematics, Third Semester.

Paper- IV & V(i):MA-634(i), (Mathematical Statistics-I) (Optional)

Note : Any five out of eight questions are to be attempted.

Probability- Set theoretic approach, Boole's inequality, Baye's theorem, Geometric probability (2 questions)

Random Variables-Distribution function, Joint probability distribution function, Conditional distribution function, Transformation of one and two dimensional Random variables (2½questions)

Mathematical Expectation- Covariance, variance of n variates, Tchebycheffs Inequality, Weak and strong Laws of large numbers. (1½ questions)

Moment Generating and Characteristic Functions and Cumulants- Central Limit theorem, Lindeberg-Levy theorem. (2 questions)

References:

1. J.Medhi; Statistical Methods, New age International (P)Ltd.
2. A.J. medhiFestschrift: Prob. &Models and Statistics, New Age International (P) Ltd.
3. Hogg (Reprint ISBN-8178086301): Introduction of Mathematical Statistics, Pearson Education.
4. J.K. Ghosh, Mathematical Statistics, John Wiley & Sons, New York.
5. J.K. Goyal & J.N. Sharma, Mathematical Statistics.
6. M.Ray& H.S. Sharma, Mathematical Statistics, Ram Prasad &Sons.
7. Gupta and Kapoor, Mathematical Statistics , S.Chand, New Delhi.
8. Goon, Gupta, Dasgupta, Fundamental of Mathematical Statistics and Applied Statistics.

M.Sc. II year Mathematics, Third Semester.

Paper- IV & V(j):MA-634(j), (Advanced Discrete Mathematics-I)(Optional)

Note: Any five out of eightquestions are to be attempted.

Formal Logic-Statements. Symbolic Representation and Tautologies. Quantifiers, Predicates and Validity. Propositional Logic.(1½ questions)

Semigroups & Monoids – Definitions and Examples of Semigroups and Monoids (including those pertaining to concatenation operation). Homomorphism of semigroups and monoids. Congruence relation and quotient Semigroups. Subsemigroup and submonoids. Direct products. Basic Homomorphism Theorem.(1½ questions)

Lattices – Lattices as partially ordered sets. Their properties. Lattices as Algebraic systems. Sublattices, Direct products, and Homomorphisms. Some Special lattices e.g., Complete, Complemented and Distributive Lattices. (2 questions)

Boolean Algebras – Boolean Algebras as lattices. Various Boolean Identities. The Switching Algebra example. Sub algebras, Direct Products and Homomorphisms. Joinirreducible elements, Atoms and Minterms. Boolean Forms and their Equivalence. Minterm Boolean Forms, Sum of Products. Canonical Forms. Minimization of Boolean Functions. Applications of Boolean Algebra to Switching Theory (using AND, OR and NOT gates). The Karnaugh Map method. (3 questions)

References

1. J.P. Tremblay & R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, McGraw-Hill Book Co., 1997.

2. J.L. Gersting, Mathematical Structures for Computer Science, (3rd edition), Computer Science Press, New York.

3. S. Wiitala, Discrete Mathematics-A Unified Approach, McGraw-Hill Book Co.4. C.L. Liu, Elements of Discrete Mathematics, McGraw-Hill Book Co.

M.Sc. II yearMathematics, Third Semester.

Paper- IV & V(j):MA-634(k), (Wavelet Analysis-I)(Optional)

Note: Any five out of eight questions are to be attempted.

Unit I Fourier Series :

Motivation and Definition, Fourier Series over the interval of length 2π, complex form of a Fourier series, Convergence of Fourier Series, Riemann Lebesgue Lemma,Convergence at a point and uniform convergence. (2 questions)

Unit II Fourier Transform :

Motivation and Definition, Basic properties, Fourier Transforms in $L^1(\mathbf{R})$ and $L^2(\mathbf{R})$, Poisson summation formula, Sampling theorem, Heisenberg's uncertainty principle, Discrete Fourier Transform,Fast Fourier Transform. (2 questions)

Unit III Time Frequency Analysis and Wavelet Transform :

Motivation and Definition, Gabor transform, Continuous wavelet transform, Basic properties of Wavelet Transform, Discrete Wavelet Transform. (2 questions)

Unit IV Multiresolution Analysis and Construction of Wavelets :

Definition and examples, Construction of Mother Wavelet, Orthonormal spline wavelets, Construction of compactly supported Wavelets, Mallat's algorithm. (2 questions)

Books Recommended :

1. K. Ahmad and F.A. Shah: Introduction to wavelets with Applications, Real World Education Publishers, New Delhi (2013).
2. G. Bachmann, L. Narici and E. Beckenstein: Fourier and Wavelet Analysis, Springer-Verlag (1999).
3. C.K. Chui: An Introduction to Wavelets, Academic Press, New York (1992).
4. I. Daubeschies: Ten Lectures on Wavelets, SIAM, Philadelphia, PA, USA (1992).

M.Sc. II year Mathematics, Third Semester.

Paper -VI : MA-535

Presentation and Viva-Voce-III

A : Viva and Presentation of assigned / selected problem /topic by each student in each of the earlier five papers to be evaluated internally throughout the semester

Total Marks : 6x5=30

B :Viva –Voce concerning all other five papers of the semester to be evaluated externally

Total Marks : 70

M.Sc. II year Mathematics, Fourth Semester.

Paper I : MA – 641, (Functional Analysis-II)

Note : Any five out of eight questions are to be attempted .

Complex linear spaces and normed linear spaces, dual spaces, dual of lp, Co and C, Reflexive spaces, weak sequential compactness. Compact Operators, Solvability of linear equations in Banach spaces.(2 questions)

Inner product spaces. Hilbert spaces. Orthonormal sets. Bessel's inequality. Complete orthonormal sets and Parseval's identity. Structure of Hilbert spaces. Projection theorem. Reisz representation theorem. Adjoint of an operator on a Hilbert space. Reflexivity of Hilbert spaces. Self adjoint operators. Positive projection, normal and unitary operators. (6 questions)

References :

1. G. Bachman and L. Narici, Functional Analysis, Academic Pres, 1966.
2. C. Goffman and G. Pedrick, First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.
3. P. K. Jain, O. P. Ahuja and Khalil Ahmad, Functional Analysis, New Age International (P) Ltd. & Wiley Eastern Ltd., New Delhi, 1997.
4. K. K. Jha, Functional Analysis, Student's Friends, 1986.
5. E. Kreyszig, Introductory Functional Analysis with Applications, John Wiley & Sons, New York, 1978.
6. B. K. Lahiri, Elements of Functional Analysis, The World Press Pvt. Ltd., Calcutta, 1994.
7. B. Chaudhary and Sudarsan Nanda, Functional Analysis with Applications, Wiley Eastern Ltds., 1989.
8. B. V. Limaye, Functional Analysis, Wiley Eastern Ltd.
9. A.H. Siddqui, Functional Analysis with Application, Tata Mac Graw Hill Publishing Company Ltd., New Delhi.
10. A. Wilansky, Functional Analysis, Blasidell Publishing Co., 1964.
11. K. Yosida, Functional Analysis. 3rd edition, Springer-Verlag, New York, 1971.
12. G. F. Simmons, Introduction to Topology and Modern Analysis. McGraw Hill Book Company, New York, 1963.

M.Sc. II year Mathematics, Fourth Semester.

Paper- II :MA-642, (Integral Equations)

Note: Any five out of eight questions are to be attempted.

Definition of integral Equations and their classification. Eigen values and Eigen functions, Fredholm integral equations of second kind with separable kernels. Reduction to a system of algebraic equations. An approximate method, method of Successive Approximations, Iterative scheme for Fredholm integral equations of the second kind. Conditions of uniform convergence and uniqueness of series solution. Resolvent kernel and its results. Application of iterative scheme to Volterra integral equations of the second kind. (3½ questions)

Classical Fredholm theory, Fredholm Theorems. (1½ question)

Symmetric kernels, complex Hilbert space. Orthonormal system of functions. Fundamental properties of eigen values and eigen functions for Symmetric kernels. Expansion in eigen function and bilinear form. Hilbert-Schmidt theorem and some immediate consequences. Solutions of integral equations with symmetric Kernels. (3 questions)

References:

1. R.P. Kanwal, Linear Integral Equation - Theory and Techniques, Academic Press, New York. 1971.
2. S.G. Mikhlin, Linear Integral Equations (Translated from Russian) Hindustan Book Agency,1960.
3. William Vernon Lovitt, Linear Integral Equations.

M.Sc. II year Mathematics, Fourth Semester.

Paper – III: MA-643, (Fluid Dynamics)

Note: Any five out of eight questions are to be attempted.

Kinematics - Lagrangian and Eulerian methods. Equation of continuity. Boundary surfaces. Stream lines, Path lines and stream lines. Velocity potential. Irrotational and rotational motions. Vortex lines. (2 questions)

Equations of motion – Lagrange's and Euler's equations of motion. Bernoulli's Theorem.

Equation of motion by flux method. Equations referred to moving axes. Impulsive actions.

Stream function. Irrotational motion in two-dimensions. Complex velocity potential Sources, sinks, doublets and their images. Conformal mapping. Milne-Thomson circle Theorem. (3 questions)

Two-dimensional irrotational method produced by motion or circular, co-axial and elliptic cylinders in an infinite mass of liquid. Kinetic energy of liquid. Theorem of Blasius. Motion of a sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere. Equation of motion of a sphere. Stoke's stream function. Introduction of vortex and wave motions. (3 questions)

References:

1. A. S. Ramsey, Dynamics Part II, The English Language Book Society and Cambridge University Press, 1972.
2. F. Chorlton, Text book of fluid Dynamics, C. B. S. Publishers, Delhi, 1985.
3. G. K. Batchelor, An Introduction to Fluid Mechanics, Foundation Books, New Delhi, 1994.
4. L. D. Landau and E. M. Lipschitz, Fluid Mechanics, Pergamon Press, London, 1985.
5. S. W. Yuan, Foundations of Fluid Mechanics. Prentice Hall of India, Pvt. Ltd., New Delhi, 1976.

M.Sc. II year Mathematics, Fourth Semester.

Paper- IV & V (a):MA-644(a), (Advanced Fluid Dynamics-II) (Optional)

Notes: Any five questions are to be attempted out of eight. .

Boundary Layer Theory : Prandtls' boundary layer theory and its importance, Boundary layer Thickness. Displacement Thickness, Momentum thickness, Energy Thickness, Drag and lift. The boundary layer equation in two dimensional flow. The boundary layer flow over a flat plate. Determination of shearing stress and boundary layer thickness. Karman's integral equation, Application of the boundary layer in absence of pressure gradient. Application of the Karman's integral equation to boundary layer with pressure gradient : Karman-Pohlhausen method. (2 questions)

Magnetohydrodynamics : Basic equations of inviscid and viscous magnetohydrodynamics. The Abfven wave. Effect of finite conductivity on hydrodynamic waves. The equation of incompressible magneto-hydrodynamic flow, Parallel steady flow. One dimensional steady viscous flow. Hartman flow, Magnetohydrodynamic characteristic equations. (2 questions)

Properties of fast, slow, transverse and entropy waves. One dimensional wave propagation Contact surfaces and transverse simple waves. Fast and slow simple waves. (2 questions)

Magnetohydroynamic shock waves. Shock waves in non-conducting gas with finite viscosity and Thermal conductivity MHD effect in stock formation. (2 questions)

References:

1. Allen Jeffery – Magnetohydrodynamics (Oliver & Boyd)
2. P. C. Kendell and C. Plumton – Magnetohydrodynamics with hydrodynamics – Vol 1 (Pergamon Press).
3. F. Chorlton – A Text Book of Fluid Dynamics.
4. M. D. Raisinghania& R.S. Agarwal – Advanced Hydrodynamics & Fluid Dynamics.

M.Sc. II year Mathematics, Fourth Semester.

Papers- IV & V (b) :MA- 644(b), (Approximation Theory-II) (Optional)

Notes: Any five questions out of eight are to be attempted.

Trigonometric approximation, Jackson integral, Jackson theorem, Inverse theorems of trigonometric approximation, Favard's theorems. (2 questions)

Improvements of estimates. Approximation by algebraic polynomials. Approximation spaces (1½ questions)

Influence of endpoints in polynomial approximation. Local inequalities for polynomials. Properties of Jackson operators Pn.m(f). (1½ questions)

Simultaneous approximation of functions and their derivatives, Estimates for function and derivative approximants. (1½ questions)

Brudnyi's theorem, Inverse theorems. Approximation spaces for algebraic polynomials (1½ questions)

References:

1. R. A. Devore and G. G. Lorentz. Constructive Approximation, Springer – Verlag, 1993.
2. M. J. D. Powell : Aproximation Theory and methods. CambridgeUniversity Press, 1981.
3. G. G. Lotentz. Approximation of functions, First ed. Holt Rinchart and Winston, New York, 1966.
4. E. W. Cheney, Introduction to Approximation theory, AMS/Chelsea publication, 1968.

M.Sc. II year Mathematics, Fourth Semester.

Paper- IV & V (c):MA-644(c), (General Relativity-II & Cosmology) (Optional)

Note: Any five questions are to be attempted selecting at least one from each section.

Energy – momentum tensor of perfect fluid, Schwarzschild internal solution, Boundary conditions. Energy momentum tensor of an electromagnetic field, Einstein-Maxwell equations, Reissner-Nordstrom solution. (2 questions)

Cosmology

Mach's Principle ,Einstein's modified field equations with cosmological term, Static cosmological models of Einstein and De-Sitter, their derivation, properties and comparison with the actual universe. (2½ questions)

Hubble's law. Cosmological principles. Weyl's postulate, Derivation of Robertson-Walker metric. Hubble and deceleration parameters. Redshift : Redshift versus distance relation, Angular size versus Redshift relation and source counts in Robertson-Walker space-time. Friedmann models. Closed and open universes. Einstein-de-Sitter model. Particle and event horizons. (3½ questions)

References:

1. H. Stephani, General Relativity-An Introduction of the theory of the gravitational field, Cambridge University Press, 1982.
2. S. R. Roy and Raj Bali, Theory of Relativity, Jaipur Publishing House, Jaipur, 1987.

M.Sc. II year Mathematics, Fourth Semester.

Paper- IV & V (d):MA-644(d), (Information Theory-II) (Optional)

Note: Any five out of eight questions are to be attempted.

Information functions, the fundamental equation of information, information functions continuous at the origin, nonnegative bounded information functions, measurable information functions and entropy Axiomatic characterizations of the Shannon entropy due to Tverberg and Leo. The general solution of the fundamental equation of information Derivations and their role in the study of information functions. (3½ questions)

The branching property. Some characterizations of the Shannon entropy based upon the branching property. Entropies with the sum property. The Shannon inequality. Subadditive, additive entropies. (2½ questions)

The Renji entropies, Entropies and mean values, Average entropies and their equality, optimal coding and the Renji entropies. Characterization of some measures of average code length. (2 questions)

References:

1. R. Ash. Information Theory, Interscience Publishers. New York, 1965.
2. F. M. Reza, An Introduction of Information Theory, McGraw Hill Book Company Inc., 1961.
3. J. Aczel and Z. Daroczy. On measures of information and their characterizations. Academic Press. New York.

M.Sc. II year Mathematics, Fourth Semester.

Paper- IV &V(e) : MA-644(e), (Mathematics of Finance and Insurance-II) (Optional)

Note: Any five out of eight questions are to be attempted.

The Black – Scholes Option Pricing Model using no arbitrage approach, limiting case of Binomial Option Pricing and Risk-Neutral probabilities. (1 question)

The American Option Pricing Extended Trading Strategies. Analysis of American Put Options early exercise premium and relation to free boundary problems. (1 question)

Concepts from Insurance : Introduction , The Claim Number Process. The Claim size Process solvability of the Portfolio : Reinsurance and Ruin Problem. (1 question)

Premium and Ordering of Risks – Premium calculation Principles and Ordering Distributions. Distribution of Aggregate Claim Amount, Individual and Collective Model : Compound Distribution : Claim Number of Distributions : Recursive Computation Methods : Lundbert sounds and Approximation by Compound Distributions. (3 questions)

Risk Processes – Time Dependent Risk Models : Poisson Arrival Processes. Ruin Probabilities and bounds. Asymptotics and Approximation.

Time Dependent Risk models – Ruin Problems and Computations of Ruin Function : Dual Queuing Model : Risk Models in Continuous Time and Numerical Evaluation of Ruin Functions.

(2 questions)

References :

1. John, C., Hull Options, Futures and other Derivatives. Prentice Hall of India Private limited.
2. Sheldon M. Ross, An Introduction of Mathematical Finance, Cambridge University, Press.
3. Saliah, N. Neftei, An Introduction to the Mathematics of financial derivatives, Academic Press, Inc.
4. Robert, J. Elliott and P. Ekkehard Kopp, Mathematics of Financial Markets, SpringerVerlag, New York, Inc.

5. Robert C.Merton, Continuous – Time Finance, Basil Blackwell.
6. C. D. Daykin, T. Pentrikamen and M. Pesonen, Practical Risk theory for Actuaries, Chapman & Hall.
7. Tomasz, Rolski, Hanspter, Schmidli, Volker Schmidt and JozefTengels. Stochastic processes for Insurance and Finance, John Wiley & Sons Limited.

M.Sc. II year Mathematics, Fourth Semester.

Paper -IV & V(f): MA-644(f), (Non-Linear Programming-II)(Optional)

Note: Any five questions out of eight are to be attempted.

- Duality in non-linear programming - Weak duality theorem. Wolf's duality theorem, Strict converse duality theorem, the Hanson-Hoard strict converse duality Theorem, Unbounded dual Theorem. Duality in quadratic and Linear Programming. (2 questions)
- Quasi convex, strictly quasi convex and pseudo convex functions, differentiability properties, Strictly quasi-convex and strictly quasi concave functions.Karmardian Theorem, Global minimum (Maximum). (1½ questions)
- Pseudo convex and pseudo concave functions, Relationship between pseudo convex functions and strictly quasi convex functions. Differentiable convex functions and pseudo convex functions. (1½ questions)
- Optimality and duality for generalized convex and concave functions – sufficient optimality theorem. Generalized Kuhn Tucker sufficient optimality theorem, Generalized Fritz John stationary point necessary optimality theorem, Kuhn-Tucker necessary optimality conditions under the weak constraint qualifications Duality. (1½ questions).
- Optimality and duality in the presence of non-linear equality constraints-sufficient optimality criteria. Minimum principle, necessary criteria : X^0 not open Minimum principle, necessary optimality theorem, Fritz, John and Kuhn-Tucker stationary point necessary optimality criteria X^0 open. Duality with non-linear equality constraints. (1½ questions).

References:

1. O. L. Mangasatian, Non-linear Programming, McGraw Hill, New York.
2. Mokhtar, S. Banzarra and C. M. Shetty, Non-linear programming, Theory and Algorithms. Wiley, New York.
3. MordeccaiAvriel, Non-Linear Programming, Analysis and Methods. Prentice Hall Inc. Englewood Cliffs, New Jersey.
4.

M.Sc. II year Mathematics, Fourth Semester.

Paper -IV & V(g):MA-644(g), (Riemannian Geometry-II) (Optional)

Note: any five out of eight questions are to be attempted.

1. **Sub-Manifold and Hypersurface of Vn:**

   Sub- manifold and Hyper surfaces of Riemannian manifold Vn. Normals, tangent space, tensor differentiation, Gauss formulae, Normal Curvature and Torsion, Weingarten formulae. Totally Geodesic subspaces, Asymptotic direction, Meunier's theorem, principal curvatures and principal directions, lines of curvature, umbilical subspaces, mean curvature, Maniardi-Codazzi equations, Gauss characteristics equation. (4 questions)

2. **Sub-Spaces and Hypersurface of an Euclidean space En:**

   Hyperplanes, Hyperspheres, Hyperquadries, sub-spaces and hypersurfaces, Jachimsthal's theorem. (2 questions)

3. **Lie Derivation in Vn:**

   Infinitesimal points transformations, Lie derivative of scalars, vectors and tensor, killing equation, Lie Derivatives of Christoffel symbols, Motion, Translation, affine motion and conformal Motion. (2 questions)

References :

1. MISHRA, R.S., A Couses in Tensor with Application to Riemannian Geometry, Pothishakka Pvt. Ltd. Allahabad. 1965.
2. Weatherburn, C.E.: An Introduction to Tensor Calculus and Riemannian Geometry.Cambridge University Press, London, 1942 and Radha Publishing House, Calcutta, India Ed. 1995.
3. Yano, K.,The Theory of Lie Derivatives and its Applications, North Holland Publishing Co.,Amesterdam, 1957.
4. Eisenhart: Riemannian Geometry, Princeton University Press, New jersey, 1927
5. Schouten, J.A.: Ricci Calculus An Introduction to Tensor Analysis and its Geometrical Applications, Springer Verlag, Berlin, II Ed. 1954.
6. Willmore, T.J.: An introduction to Differential Geometry, Oxford university Press,London 1954 and 11th Indian Ed., New Delhi, 1993.

M.Sc. II year Mathematics, Fourth Semester.

Paper -IV & V(h): MA-644(h), (Space Dynamics-II) (Optional)

Note: any five out of eight questions are to be attempted.

Motion of the moon-the perturbing force, perturbations of Keplerian elements of the Moon by the Sun. (1½ questions)

Flight mechanics- Rocket performance in a vacuum. Vertically ascending paths . Gravity twin trajectories. Multi stage rocket in vacuum. Definitions pertinent to single stage rockets. (1½ questions)

Performance limitations of single stage rockets, definitions pertinent to multi stage rockets, Analysis of multistage rockets including gravity. (2½ questions)

Rocket performance with Aerodynamic forces. Short range non-lifting missiles, Ascent of a sounding rocket. Some approximate performance of rocket- powered air- craft. (2½ questions)

References :

1. J.M.A. Danby, fundamentals of Celestial Mechanics, The Macmillan Company.1962.
2. E.Finlay, Freundlich ,Celestial Mechanics, The Macmillan Company,1958.
3. Theodore E.Sterne, An Introduction of Celestial Mechanics, Intersciences Publishers,Inc.1960.
4. Arigelo Miele, Flight Macchanics-Vol1, theory of flight paths, Addison-wesley publishing company, Inc.,1962.

M.Sc. II year Mathematics, Fourth Semester.

Paper - IV & V(i): MA-644(i), (Mathematical Statistics-II) (Optional)

Note: Any five out of eight questions are to be attempted.

Distributions relationship with each other, distribution of their sum, difference, product, quotient etc. (a) Binomial, Poisson, Negative Binomial, Geometric, Pascal's Polya's Hypergeometric distributions, Multinomial power series and discrete uniform, compound binomial and Poisson distributions. (2 questions)

Log-normal, log-normal, Gamma, Beta, Exponential, Bivariate Normal, Laplace, Weibul, Cauchy and Pearson's distributions. (1½questions)

Derivation of Chi-square distributions, Non central chi-square distribution. (1 question)

Test of significance. Distribution Function-of t. F and z test of significance. (1 question)

Theory of estimation principle of maximum likelihood, properties of maximum likelihood estimators. (1 question)

Analysis of variance -analysis of variancein one way and two ways classification. (1½ questions)

References:

1. J.Medhi; Statistical Methods, New age International (P)Ltd.
2. A.J. medhiFestschrift: Prob. &Models and Statistics, New Age International (P) Ltd.
3. Hogg (Reprint ISBN-8178086301): Introduction of Mathematical Statistics, Pearson Education.
4. J.K. Ghosh, Mathematical Statistics, John Wiley & Sons, New York.
5. J.K. Goyal & J.N. Sharma, Mathematical Statistics.
6. M.Ray& H.S. Sharma, Mathematical Statistics, Ram Prasad &Sons.
7. Gupta and Kapoor, Mathematical Statistics , S.Chand, New Delhi.
8. Goon, Gupta, Dasgupta, Fundamental of Mathematical Statistics and Applied Statistics.

M.Sc. II year Mathematics, Fourth Semester.

Paper - IV & V(j):MA-644(j), (Advanced Discrete Mathematics-II)(Optional)

Graph Theory – Definition of (undirected) Graphs Paths, Circuits; Cycles, and Subgraphs. Induced Subgraphs. Degree of a vertex. Connectivity. Planar Graphs and their properties. Trees, Euler's Formula for connected Planar Graphs. Complete and Complete Bipartite Graphs. Kuratowski's Theorem (statement only) and its use. Spanning Trees, Cut-sets, Fundamental Cut-sets, and Cycles, Minimal Spanning Trees and Kruskal's Algorithm. Matrix Representations of Graphs.(3 questions)

Euler's Theorem on the Existence of Eulerian Paths and Circuits. Directed Graphs. In degree and Out degree of a vertex. Weighted undirected Graphs. Dijkstra's Algorithm. Strong Connectivity and Warshall's Algorithm. Directed Trees. Search Trees. Tree Traversals.(2 questions)

Introductory Computability Theory – State Machines and their Transition Table Diagrams. Equivalence of Finite State Machines. Reduced Machines. Homomorphism. Finite Automata. Acceptors. Non-deterministic Finite Automata and equivalence of its power to that of Deterministic Finite Automata. Moore and Mealy Machines.(2 questions)

Turing Machine and Partial Recursive Functions.(1 question)

References:

1. J.P. Tremblay & R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, McGraw-Hill Book Co., 1997.

2. J.L. Gersting, Mathematical Structures for Computer Science, (3^{rd} edition), Computer Science Press, New York.

3. Seymour Lipschutz, Finite Mathematics (International edition 1983). McGraw-Hill Book Company, New York.

4. S. Wiitala, Discrete Mathematics-A Unified Approach, McGraw-Hill Book Co.

5. J.E. Hopcroft and J.D. Ullman, Introduction to Automata Theory, Languages & Computation, Narosa Publishing, House.

6. C.L. Liu, Elements of Discrete Mathematics, McGraw-Hill Book Co.

7. N. Deo, Graph Theory with Applications to Engineering and Computer Sciences, Prentice hall of India.

M.Sc. II yearMathematics, Third Semester.

Paper- IV & V(j):MA-644(k), (Wavelet Analysis-II)(Optional)

Note: Any five out of eightquestions are to be attempted.

Unit I Wavelet Construction:

Biorthogonal Wavelets, Wavelets in Several variables, Multiwavelets, Wavelets Frames. (2 questions)

Unit II Wavelet Packets :

Construction of Wavelets packets, Certain results on wavelet packets, Band limited wavelet packets, characterisation of wavelets packets. (2 questions)

Unit III Convergence of Wavelet Packet Series :

Pointwise Convergence of wavelet packet series, convolution bonds and Convergence of Wavelet packet series. (2 questions)

Unit IV Applications:

Statistics, Neutral networks, turbulence, Medicine, Economics and finance, Signal and Image processing. (2 questions)

Books Recommended :

1. K. Ahmad and F.A.Shah: Introduction to wavelets with Applications, Real World Education Publishers, New Delhi (2013).
2. E. Hernadez and G. Weiss: A First Course on Wavelets, CRC press, New York, (1996).
3. I. Daubechies: Ten Lectures on wavelets, SIAM, Philadelphia, PA, USA (1992).
4. K.Ahmad and Abdullah: Wavelet packets and Applications, Springer (2017).

M.Sc. II year Mathematics, Fourth Semester.

Paper - VI : MA-645

Presentation and Viva-Voce-IV

A :Viva and Presentation of assigned / selected problem /topic by each student in each of the earlier five papers to be evaluated internally throughout the semester

Total Marks : 6x5=30

B :Viva –Voce concerning all other five papers of the semester to be evaluated externally

Total Marks : 70